Raytracing in the Compensation of the Peripheral Optics of the Eye

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Abstract

Background: Many people with a visual impairment have only peripheral vision. However, there is limited knowledge of the peripheral optics of the eye and only some measurements are available in this field. *Methods:* We simulated the paths of peripheral rays through the eye by means of raytracing. Five programs were compared. The OSLO raytracing software proved to be not only the best one in these circumstances but we also found it very well suited to our purpose. Remaining uncertainties are entirely due to a lack of input data about the peripheral part of the optical system of the eye. We designed compensatory optics on the basis of the test results. *Results:* Lenses have been manufactured in accordance with the calculations made by the program for angles of incidence of 20, 40, and 60 degrees. The lenses are high compensation astigmatic lenses. The results of perimeter examinations of changes in peripheral vision using attachment optics were inconclusive, while tests of the lenses as attachments in front of a fundus camera produced successful preliminary results. *Conclusion:* The next step is to test peripheral vision compensatory optics in traffic situations (driving simulator). At the same time attempts are being made to find methods and instruments for measuring the peripheral optics of the eye.

Keywords: astigmatism, central scotoma, raytracing, macula degeneration, peripheral vision.

1. Introduction

In various contexts, it has been suggested that improvements to peripheral optics would not improve human peripheral vision. The reasons put forward have been that there are too few cones outside the fovea[1] or that aliasing, for example, would undermine the conditions for clear vision.[2]

However, seeing on a daily basis patients who are able to use eccentric fixation successfully has made us doubt the relevance of such arguments. Our aim is thus to develop the best possible peripheral optics and subsequently study their effect, if any, on people with central scotoma, e.g. macula degeneration, as well as on people with normal vision in situations where peripheral vision is particularly important (e.g. in traffic). A more detailed description of this project is available at `http://www.certec.lth.se/english/widesight`

The fact that optical imaging in the human eye is only one of the parameters controlling human vision means that the connection between improved vision and compensatory optics is not an obvious one. For instance, sometimes there are major differences between the compensatory optics providing the best vision according to the patient and the best compensatory optics measured by the optometrist. This is particularly interesting in the case of those patients for whom a difference can be observed between the objective and the subjective methods. For example, it is common for patients with central scotoma to want more powerful compensation than that predicted by an objective method such as retinoscopy when correcting refraction errors. Retinoscopy cannot be carried out satisfactorily for angles exceeding 10-15 degrees to the optical axis. If the patient is using parts of the retina outside this area, it is likely that retinoscopy or other objective methods will provide incorrect values. [3]

Rough estimates based on models of the eye tell us – just as the patients do – that in order to ensure better acuity stronger refraction is required in the peripheral area than in the central area. However, mental calculations cannot provide any reasonable estimates of other aberrations, such as astigmatism and, at present, no reliable measuring instruments are available for studying the refraction of peripheral rays. We were therefore obliged to use raytracing programs in calculating compensatory optics for oblique rays. Our intention was subsequently to test these optics on, for example, people with central scotomas.

2. Raytracing

Raytracing, i.e computer-simulated ray paths, has existed for 30 years. It has become increasingly sophisticated and is now used as a matter of course when designing large optical systems. However, until recently there was essentially no raytracing applied to the human eye. Nonetheless, graphical raytracing programs could have many applications in vision contexts. It could be applied to the whole spectrum from research to direct patient contacts. A raytracing program could, for example, be an excellent educational tool for optometrists: at the office, individual data could quickly be entered into the system to give the patient a clear understanding of the impact of his visual impairment and the effectiveness of various types of compensatory optics.

There has been, and still is, a considerable gap between ophthalmology and ophthalmic surgery on the one hand and basic, applied optics on the other. The fact that raytracing systems for the eye were developed at all can be directly attributed to the need for more differentiated information about the effects of changes to the curvature of the cornea in connection with excimer laser operations.[4] However, not a great deal has been published on raytracing used in a vision context. When searching Medline in March 1999 for articles containing the keyword “raytracing” we got only three hits for the period between 1996 and 1998 and five between 1993 and 1995.

Works published on this subject have often been collaborations between optometrists and physicists. Our group is no exception in that regard, but it also includes an experienced user of a large commercial raytracing program and a creator of applications with extensive software experience. The interaction between him and the optometrist in the group has generated many new ideas (for him as well as us.) In terms of methodology, it is relevant to compare our reasoning with the line of argument in *The Reflective Practitioner* by the Donald Schön.[5]

In the first instance, we were not necessarily looking for a raytracing system that would provide strong graphical support to the user. Our main requirement was that the system should be capable of working with peripheral rays (up to 90 degrees) and of handling a complex eye model (including a gradient index lens), and that it should comprise optimisation functions. The known Gullstrand schematic eye model and similar models do not provide sufficient data to satisfy our purposes. Instead, we used the data published in 1997 by Hwey-Lan Liou and Noel A. Brennan.[6]

3. Various Raytracing Programs

It is far from obvious that one will find a program that is usable for the peripheral optics of the eye by searching among existing raytracing programs. Software can rarely be used for areas other than those for which is was created since the intentions of the creator of the application are consciously or subconsciously built into the software and its optimisation. Accordingly, we were not successful in our attempts at using eye raytracing programs created on the basis of *specific* interests which were different from our own.   However, we used a larger, more *general* raytracing system (OSLO) with considerable success. An overview of various raytracing programs is provided below.

1. The SGP program

Initially, we used a fairly old raytracing program created by a member of our group, Sven-Göran Petterson, which we applied to the eye. We were able to progress relatively far with the SGP program, but the breaking point came when the program code grew so extensive that it became impossible to add new capabilities without removing others. At this point, our alternatives included recreating and modernizing the program or moving on with a newer and larger program. The fact that we chose the latter does not diminish the importance of the SGP program to this project. For example, the SGP software made us aware of the shortcomings of existing eye models, of how small errors in these models produce major deviations in peripheral optics, etc. In addition, it became obvious that the program must permit the use of a gradient index lens. Different models, such as Gullstrand and Kooijmans, were used in this program.[7,8,9]

2. The Tübingen program

The first raytracing program adapted for eye applications, which we used, was a program developed in Tübingen.[10] As described in the article, by 1996 the Tübingen group had already made a great deal of progress both in developing beamlight calculations and in finding ways of presenting the results to the layperson, for example the patient. The idea of having double ray paths and reflecting the light onto a screen for direct comparison is an attractive one.

The Tübingen group has developed a linear accommodation model. This means that all parameters having different values in the Gullstrand model are modified by a λ parameter. This parameter has the value 1 for a relaxed eye and the value 0 when the eye is focused on the near point. Accordingly, a radius R (e.g. for the first surface of the lens) can described by the following formula:

$$R_{acc} = R_{n\ddot{a}ra} + (R_{\infty} - R_{n\ddot{a}ra}) \cdot \lambda$$

where R_{acc}, $R_{¥}$,, R_{near} is the radius of the accommodated position, for a relaxed eye and for an eye focused on the near point. The Tübingen group adjusts λ to achieve the best imaging at an arbitrary object distance.

The program is written in turbo pascal. The program was not usable for our purposes (see analysis below).

3. Visual Optics Lab

In our attempts to move ahead using commercial software, we subsequently tested a beta version of the commercial Visual Optics Lab,[11] Ed Sarver, USA. Visual Optics Lab has the advantage of being designed for the human eye but it has the drawbacks that it cannot comprise a gradient index lens and that it is not primarily intended for peripheral rays.

4. Sigma

We also tested Sigma 2000 and the Sigma 2100 upgrade from Kidger Optics. These programs seemed promising since they included the possibility of using a gradient index lens. In addition, optimizations were possible. We successfully studied rays centrally through the eye but when an out-of-center cross cylinder was placed in front of the eye, the optimizations produced strange results. One problem associated with this program was that the parameters were changed automatically, outside the control of the user. Sometimes it was not possible to choose certain surfaces in certain conditions. For example, it was not possible to use an elliptical retina when the light was incident at 60º. In order to reach 60º it was necessary for the last surface to be spherical. Evidently, it was necessary to be very familiar with the program to be able to use it in such a way that it would produced reliable results. The later version (Sigma 2100) offered the attractive possibility of having a small bitmap image as an object. However, it turned out that in this version it was no longer possible to use a gradient index lens with aspheric surfaces. It is possible that this is the reason why aspheric surfaces did not produce satisfactory results in the earlier version.

5. CODE V

CODE V is the leader among commercially available general optics programs. The program is available from Optical Research Associates, Pasadena, California, USA. It is used by a number of companies and institutes around the world for designing the optics of, for example, photographic equipment, video cameras, and medical instruments. It is flexible and reliable, and it is supported by a large number of staff. The program is available for lease only and at present we do not have access to it.

6. OSLO

Our final choice was OSLO, a large general optical computer application from Sinclair Optics, USA.[12] Several versions of the program are available and the most advanced version offers more or less the same possibilities as CODE V. The program originates from the optics program at the University of Rochester, USA. The program is flexible and the fact that the user can add personal macro routines written in C means that he has almost unlimited possibilities for solving optical problems. We bought the program which is a great deal cheaper than CODE V.

One of the main reasons why OLSO afforded the best possibility of achieving good results in our field of interest is that the capacity of OSLO is so great that it does not shy away from difficulties such as handling a complex eye model – including a gradient index lens – or supplying optimised compensation lens data. However, a large application such as OSLO requires a very competent and experienced user – it is almost necessary to view the system and the user as a single unit. In our group, Bo Möller fulfils this necessary function. Our work was focused on optimising peripheral optical compensation for sharp imaging up to 60°.

4. Usability and Useworthiness

Studying the usability of raytracing programs requires a knowledge of optics as well as experience as a user of software and a creator of applications. Usability centres on whether the program solves the tasks it can perform in a simple, practical, reliable, fast, and flexible manner. The usability of a program does not necessarily tell us anything about the *useworthiness* of the specific application of the program to the *peripheral optics of the human eye* which is the aim of our work. The concept of useworthiness is analysed in detail in a doctoral thesis by Håkan Eftring, September 1999.

`http://www.certec.lth.se/english/hakan.eftring`.

We tested the programs described above and found that the OSLO general optics program was superior for our purposes. It is described in detail below, followed by shorter analyses of the other programs.

OSLO

Userfriendliness and learning time

The program is very easy to use – once you know it. However, it is a very large program. Going through a tutorial covering the basic functions takes a day or two, but mastering all the features requires years of use. A knowledge of optics is required in order to understand the terminology.

Excecution efficiency, iteration time, and transparancy.

Calculations are fast and it is easy to change the input. The built-in optimisation functions are designed in such a way that the user does not lose control of what is happening and why.

Reliability, imperfections, and annoying bugs

Few real errors. There are many users of this program, which makes debugging faster. There are some imperfections and simplifications.

Graphics, input functions, and standard Windows interface

Graphic outputting is available in a number of different forms. On the other hand, graphic inputting is not very developed. It requires mathematical formalism, e.g. wavefronts in Zernike polynomial development. However, inputting is easy and fast for the experienced user. The user interface has been redesigned and adapted to Windows.

Help functions

The Help file is extensive.

Degree of technical excellence

The program is at the leading edge. New developments in the field are quickly introduced.

Expandability and generalizability

A macro language is available, essentially based on the C programming language, which enables the user to write his own (compilable) routines for applications, diagrams, etc., which may not be included in the program. Macros can be supplied by other users and they are not limited to the applications which the creator of the software has found to be commercially viable. Successful macros created by customers are often incorporated into the program. Issues relating to the optics of the eye have not been a driving force behind the development of the software, and it is likely that there are aspects relating to the optics of the eye which have not been taken into consideration in the program. However, our requirements were met through the use of macros.

Support, service, and updates

The company provides quick and knowledgeable answers to questions, usually within 24 hours. Updated and expanded versions of the software are produced a couple of times a year. If the user finds any errors, the company tries to find a way around them until a new version has been produced. In this connection, macros are very useful. Since this is a general program, a change/addition to the program must be of general interest if it is to be incorporated into the program.

Price

The version of the OSLO software which can handle gradient index lenses is called OSLO Pro and costs $2695. Updates cost $895 per year.

Summary of the OSLO software: This program is useworthy for the purposes of studying the optics of the eye, including peripheral rays. With OSLO we have come far enough that it is no longer the software which imposes limits on what can be studied. Rather, the limiting factor is the limited availability of optical data relating to the human eye.

Other programs

None of the other four programs included in our study meets the requirement of useworthiness in terms of our objective, which is to study the peripheral optics of the human eye. The programs must have optimisation functions for optical compensation and must be capable of handling a complex eye model comprising a gradient index lens.

Initially, the SGP program proved very useworthy to us. For example, it demonstrated both the necessity of using a gradient index lens and the critical nature of the data relating to the gradient index lens. On the other hand, the program would require a thorough modernisation to be capable of carrying out calculations within a reasonable amount of time.

The basic concept of the Tübingen program – using a linear accommodation model for optimal imaging at an arbitrary object distance and presenting the result in a way which is easy to understand for the layperson – is appealing, but it cannot be expanded for our purposes.

Just like the Tübingen program, Visual Optics Lab was developed for the human eye but it has the drawbacks that a gradient index lens cannot be included in the model and that it is not possible to work with peripheral rays. The program has a superb graphical interface, making it suitable for teaching purposes.

Kidger Optics Sigma 2000 and the Sigma 2100 upgrade are capable of including a gradient index lens and of handling optimisations. The reason we decided against it was that optimisations became difficult when a cross-cylinder was placed in front of the eye. In Sigma 2100, offering the attractive possibility of having a bitmap image as an object, it was not possible to use a gradient index lens with aspherical surfaces.

Finally: in addition to OSLO there is one program which may fulfil our requirements: CODE V. It has extensive commercial applications and is probably both flexible and reliable and is supported by a large number of staff. The reason we decided against it was that it is available for lease only and that the monthly leasing fee is approximately the same as the purchase price of the OSLO PRO.

5. Our Choice of an Eye Model and Input Data for OSLO

We used data from the eye model presented by Hwey-Lan Liou and Noel A. Brennan.[13] The model is based on a large number of measurements of the eye and comprises a gradient index lens with two areas with somewhat differing refraction index variations. The model is optimised with respect to spherical aberration and chromatic aberration on the axis. In order to obtain results which correspond with the measured values of the above-mentioned aberrations, the rear surface of the cornea has been varied. Since this model is based on rays adjacent to the optical axis only it is not necessary for the peripheral imaging to be correct. Careful measurement of aberrations at large angles is probably a sensitive way of determining what the refraction index distribution in the eye lens looks like. But if the purpose is only to demonstrate that it is possible to produce compensatory optics with the aid of the optimisation routines in OSLO, it is not essential at this stage whether or not the eye model is correct. The optical eye data we used are shown in tables 1-4. Figure 1 shows the model as it is presented in the OSLO program.

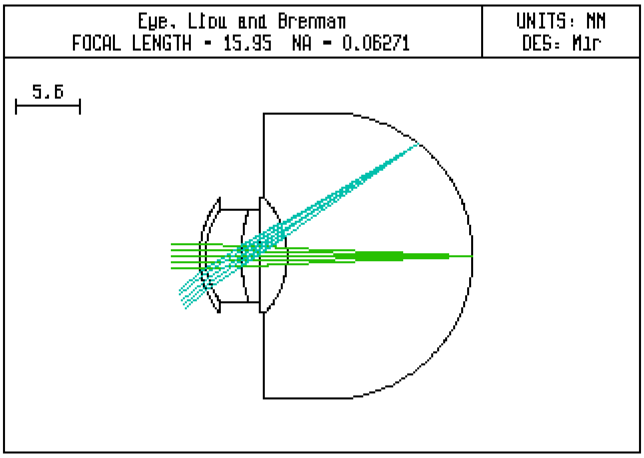


Figure 1. The model used with rays drawn at 0º and 40º.

Table 1. Lens data for the Liou and Brennan eye model.

Table 2. Gradient index data

Table 3. Refraction index

Table 4. Paraxial eye data at an angle of incidence of 60°.

Liou and Brennan describe the gradient index lens as follows:

$$n(w,z) = n_{00} + n_{01}z + n_{02}z^2 + n_{10}w^2$$

where z is along the optical axis and w ($w^2 = x^2 + y^2$) is the radial distance at right angles to the z-axis and n_{00}, n_{02} and n_{10} are the refraction index coefficients of a parabolic gradient index distribution in an unaccommodated eye. Higher order terms are set at 0. The refraction index values are shown in Tables 2 and 3.

When carrying out our polychromatic evaluations, we used 555 nm as the fundamental wavelength and 510 nm and 610 nm as side wavelengths. The weight of the fundamental wavelength was 1 while the weight of the side wavelengths was 0.5. As shown in Table 4, the eye has a total length of 23.95 mm in accordance with the value given by Liou and Brennan. When calculating a beamlight through gradient index surfaces 4 and 5 we used a step length of 0.020 mm.

We chose to carry out the investigation using a 2 mm pupil This pupil diameter is approximately the same as the size of the eye in normal daylight. One method of checking the quality of the imaging is to study spot diagrams, i.e calculating rays uniformly distributed across the pupil to the retina and studying the picture. For the eye model used, the images shown in Figs 2a-d were obtained at angles of incidence of 0, 20, 40, and 60 degrees.

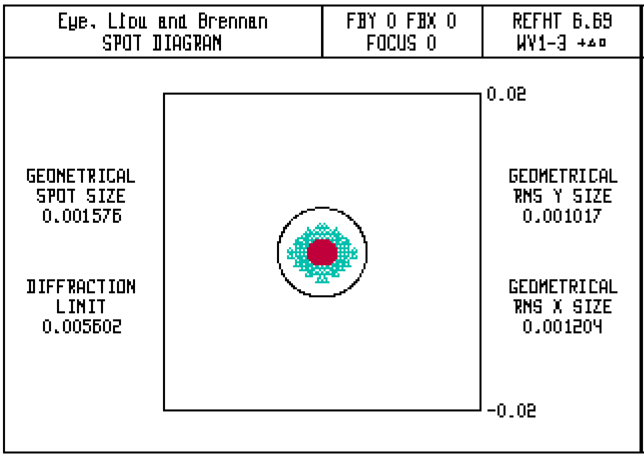


Figure 2a. Spot diagram at 0°

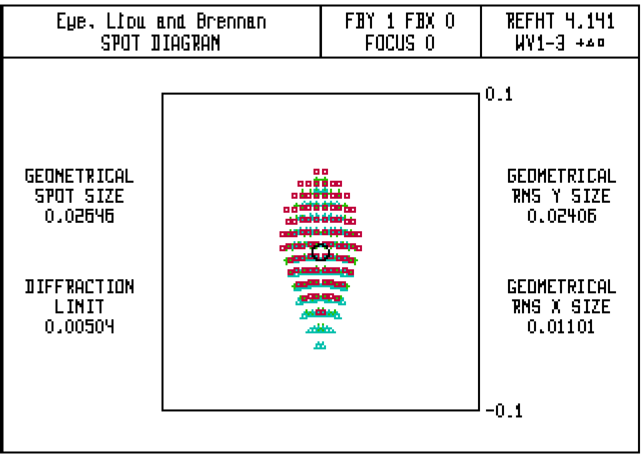


Figure 2b. Spot diagram at 20°

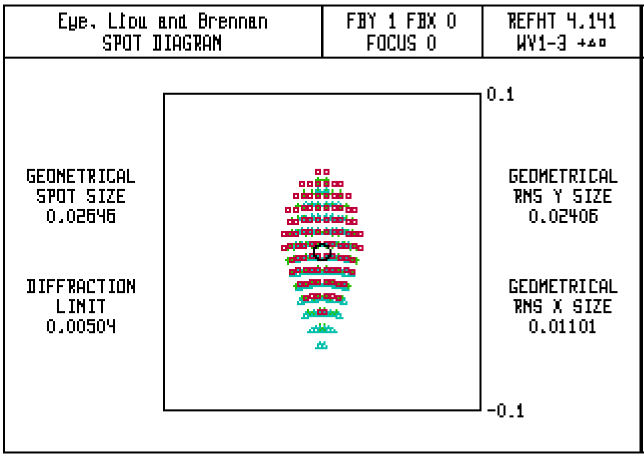


Figure 2c. Spot diagram at 40°

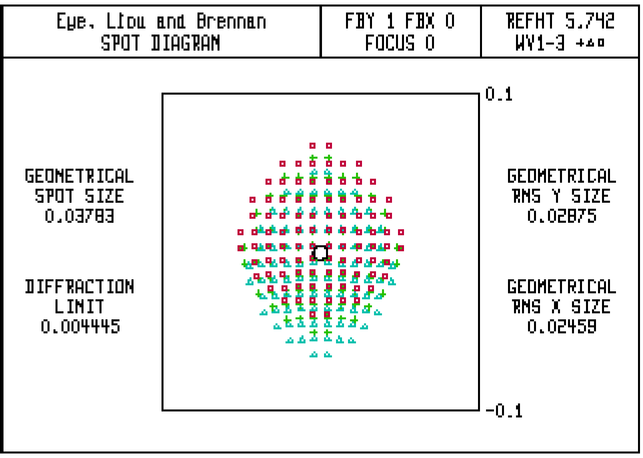


Figure 2d. Spot diagram at 60°

The oblong shape of the spot diagrams indicate that the most prominent aberration is astigmatism.

For the purpose of optimising optical compensation for the eye we chose to attempt to correct the aberrations using a simple lens consisting of two crossed cylinder surfaces. The lens was placed at a distance of 12 mm (calculated along the ray going from the correction lens through the centre of the pupil) from the cornea of the eye, perpendicular to the ray (see Fig. 3). Since astigmatism increases as the angle of incidence increases, we calculated one correction for each angle. It is assumed that the material used in these lenses is organic (CR-39) having a centre thickness of 2.2 mm.

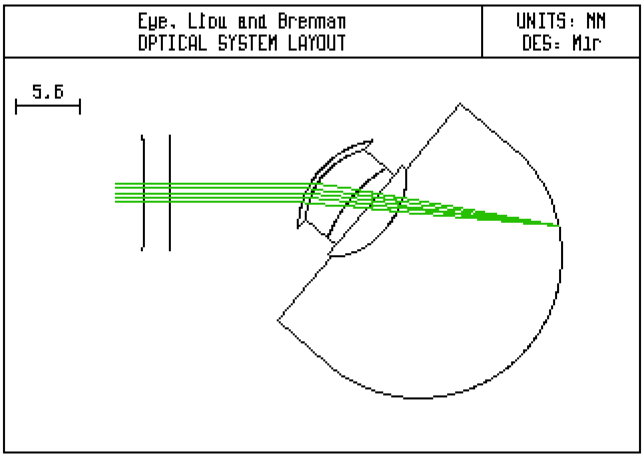


Figure 3. The eye shown with a corrective lens placed 12 mm in front of the cornea. The angle of incidence is 40° to the optical axis of the eye.

6. Results

a. Calculated Values for Compensatory Optics

The flexible optimisation routine available in OSLO proved extremely useful for calculating values for compensatory optics. We chose a number of rays in the tangential direction and a number of rays in the sagittal direction and requested that their points of impact should be made to coincide in one point as much as possible by varying the radii of the two cylinder lenses. The result for 40° is shown in Figs 4a-c. As the Figures show, the quality of the image improved significantly. The tangential and sagittal pictures were considerably closer to the retina. Now, other aberrations appeared, such as lateral colour aberration and coma. However, using a simple centred cross-cylinder lens with circular radii, it was not possible to correct these aberrations in these initial trials and, accordingly, the correction of primarily coma will be attempted later. The Airy ring is indicated in the Figures. If the aberration is negligible, it corresponds to the size of the diffraction image . The Strehl ratio is a measure of the quality of the image, indicating the extent to which the rays fall within the Airy ring, i.e. how close the image is to being defraction limited. In our calculations we achieved Strehl ratios exceeding 0.95.

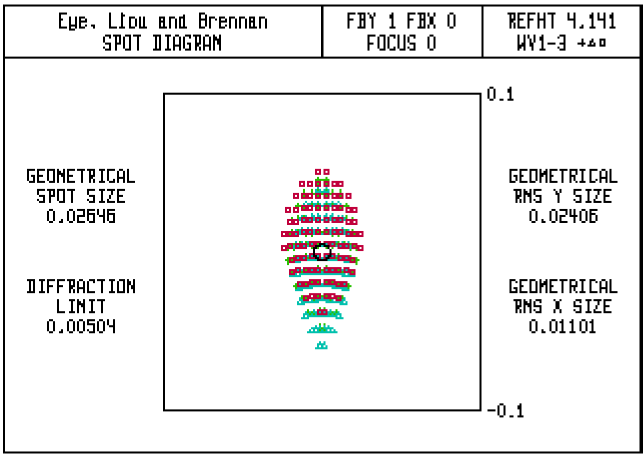


Figure 4a. At 40° without correction.

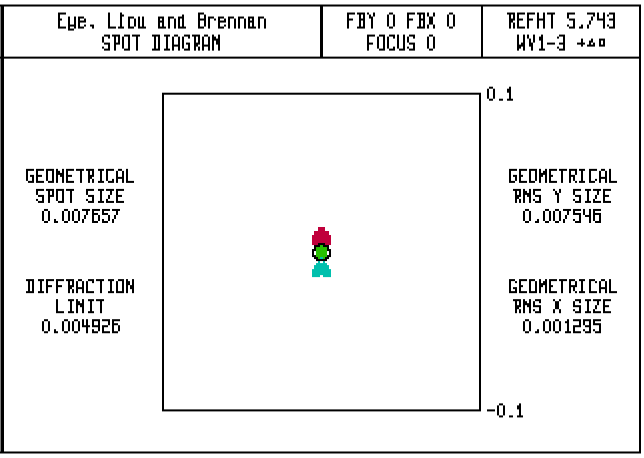


Figure 4b. With correction.

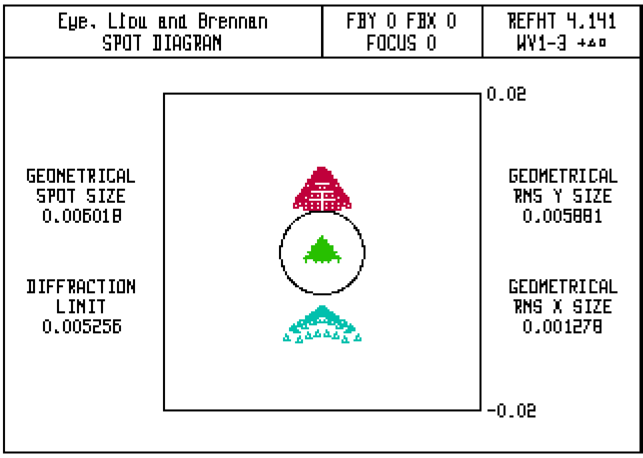


Figure 4c. Figure 4b, enlarged.

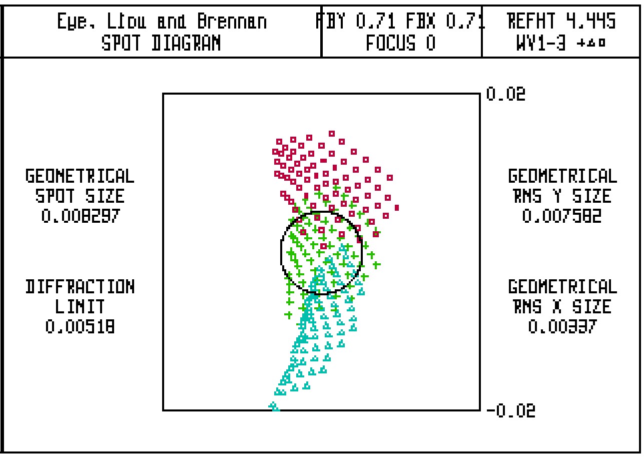


Figure 4d. Spot diagram for light around the optimal angle.

The same type of optimisations were carried out for angles of incidence of 20° and 60°. Table 5 shows the values of the lenses calculated.

Table 5. Calculated acrylic compensation lenses (n = 1.492)

Angle of incidence	y-direction radius/mm	y-direction diopters	x-direction radius/mm	x-direction diopters
20°	462.9	-1.06	1720	+0.29
40°	118.6	-4.15	358	+1.37
60°	60.89	-8.07	157	+3.14

We also studied how well the correction works for a field of vision around the angle for which the optimisation was carried out. This can be seen in Figure 4d, which shows the distribution in 5° obliquely upwards with an azimut angle of 45°. In this case, the compensation lenses are optimised for 40°. The image quality deteriorates (Strehl ratio ˜ 0.3) but is considered satisfactory to about this angle.

When taking photographs of the optic fundus, a much larger pupil (about 8 mm in diameter) is used. Imaging in 40° with this pupil is shown with and without correction lenses in Figs 5a-b and Figs 6a-b. The impact of the remaining coma is clearly visible and compensation only results in a small improvement.

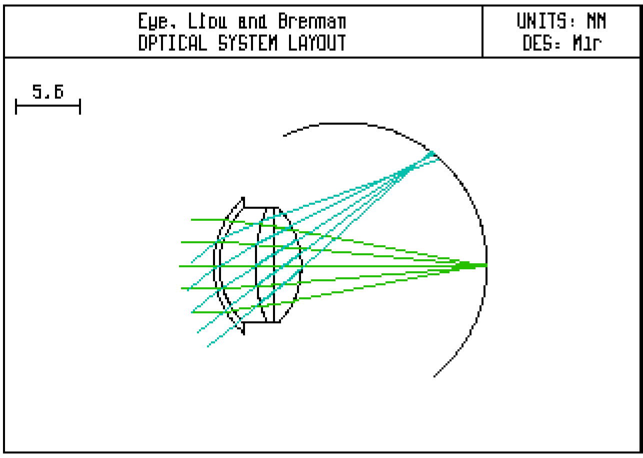


Figure 5a. Imaging with an 8 mm pupil.

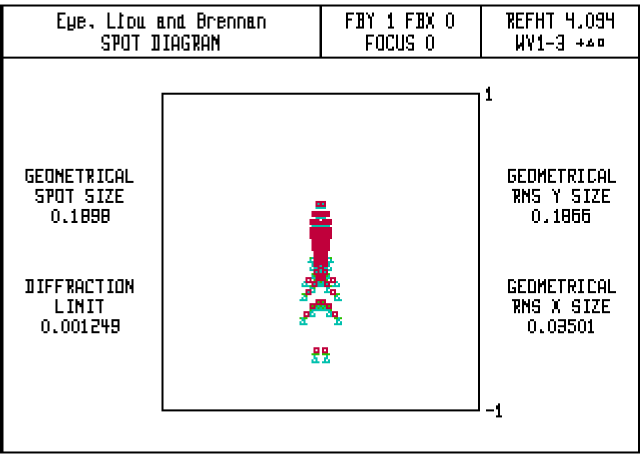


Figure 5b. Spot diagram at 40°.

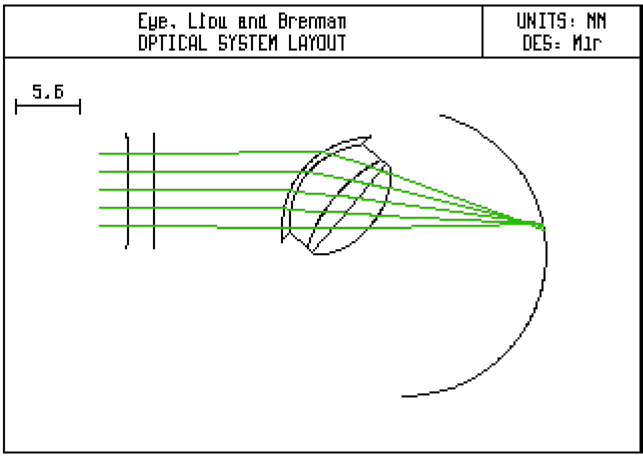


Figure 6a. Compensation at 40°.

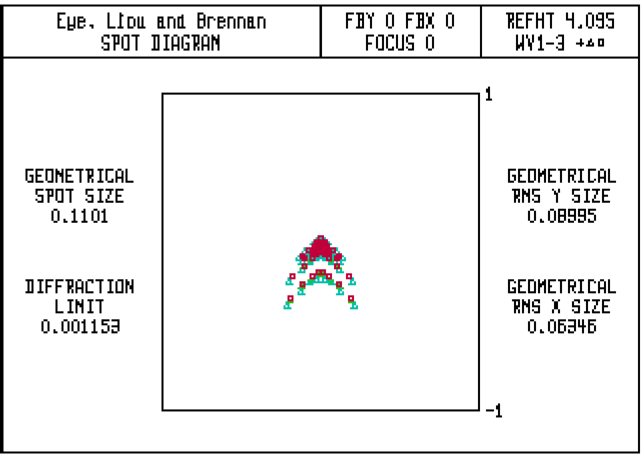


Figure 6b. Spot diagram after compensation.

b. Manufacture of Compensation Lenses

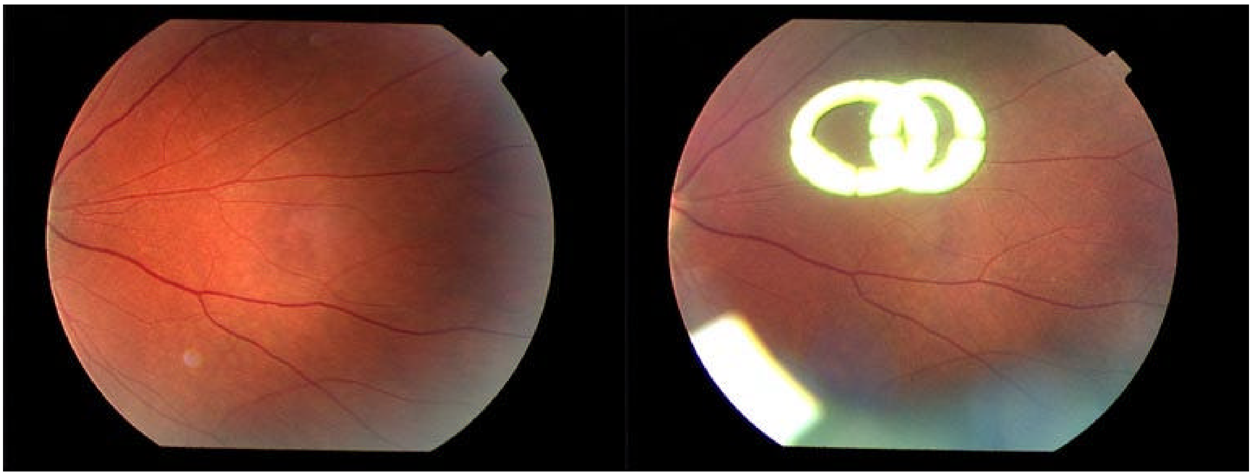
MultiLens AB in Göteborg, Sweden, manufactured the above-mentioned lenses. They are made from an organic material. They are made as a combination of a front plus-toric and a back minus-toric surface in the same lens. The above values were used in the eye glass plane for the 20, 40, and 60 degree angles.

c. Testing in Front of a Fundus Camera

Since ray paths are reversible, the task of achieving a sharp image of the retina in a fundus camera is the equivalent to the task of ensuring the sharpness of the image on the retina. We were greatly encouraged when Lijun Zhu et al published the article *Modeling human eye aberrations and their compensation for high-resolution retinal imaging*[13] in 1998. Their work is not primarily focused on improving optical imaging *in* the human eye but rather on improving optical imaging *of* the human eye, i.e. on making better fundus cameras. The group designed a computer model of an eye based on Gullstrand's six surface model modified with data concerning non-spheric surfaces with respect to the eye lens as well as the retina. Despite the fact that their raytracing use of Code V did not per se provide us with many new ideas, their approach and results are very exciting and useful.

We took photographs of the optic fundus through the compensation lenses manufactured by MultiLens. At an angle of 20 degrees, the quality of the image was so high from the start (after focusing the camera) that no difference could be observed in this way. At an angle of 60 degrees, it was so difficult to obtain a sharp image that no difference could be observed.

However, at a 40 degree angle the imaging was considerable inferior in the periphery to the right in the image compared with what it was when the compensation lens was placed in front of the eye (at a vertex distance of 12 mm). See Figs 7a and b. Both images were taken with the "best possible sharpness" of the camera under the two different conditions.



d. Testing in Perimeters

Ordinary testing of the field of vision using Goldman perimetry or computerised perimeters such as Humphrey has not produced conclusive results.

e. Planned Testing

In September 1999 initial measurements will be carried out in a driving simulator, with and without compensatory peripheral vision optics in order to establish whether there are any improvements in the detection of objects coming from the side. In these trials, we will thus test the vision of the test subjects (when standing still and when moving) as well as their reactions.

7. Discussion

In humans, sharp vision only occurs within about 5 degrees centrally. Despite the fact that many individuals with loss of central vision are obliged to use peripheral parts of the retina, optical studies of the remaining 175 degrees have so far been insufficient. Refractometers and other objective methods of evaluating the refraction of the eye provide reliable values only on or near the optical axis. Moreover, they are only designed to measure spherical and cylindrical aberrations and are not usable for other aberrations, which are dominant at large angles to the optical axis.

Raytracing software seems to be a method of calculating required compensatory optics for the eye. However, in the course of our work with raytracing be have become aware of the fact that there is very limited knowledge of the peripheral optics of the eye. Knowledge about the eye lens and its gradient index structure is particularly critical to the precision of theoretical calculations. This awareness has encouraged us in our plans to develop measuring instruments for the peripheral area.

However, our overriding goal is still to develop better compensatory optics for people with loss of central vision and (if possible and suitable) for better peripheral vision in traffic situations.

8. Conclusions

Most raytracing programs are not usable for studying the peripheral optics of the human eye. OSLO is an exception. With OSLO, we have come so far that the greatest uncertainty about the results is due to insufficient input data, i.e. the present lack of knowledge about the optical structure of the eye. It is evident that new measuring instruments are needed in order to generate improved knowledge of the peripheral optics of the eye.[14,15,16]

We have shown that it is possible to incorporate a gradient index lens[17,18] in a raytracing program, to calculate correction lenses for peripheral vision, and to manufacture a first generation of such lenses. Trials are under way, and the most positive result so far is that it has been possible to obtain a better image in an fundus camera of a peripheral part of the retina.

The next step in the trials of the first generation lens is to test the peripheral vision compensatory optics in traffic situations (driving simulator).

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